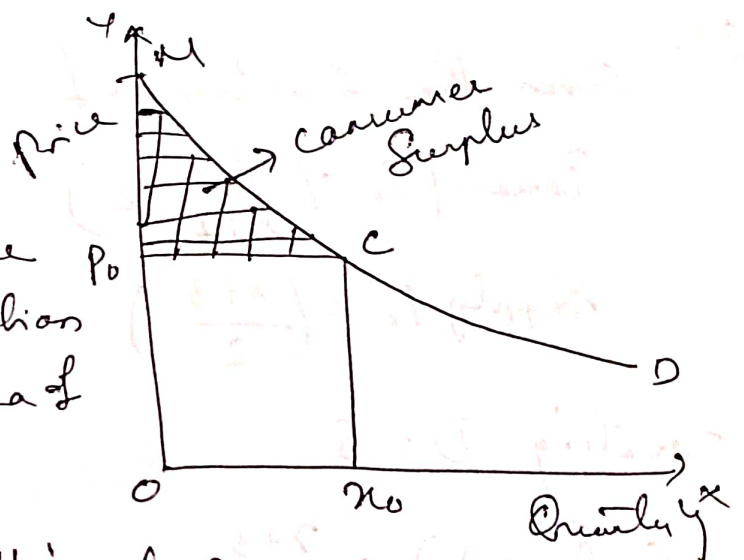


Consumer's Surplus

Given the demand function with price P_0 and quantity demanded x_0 , consumer surplus is total area of the curve below demand function from 0 to x_0 minus the area of the rectangle $OP_0C x_0$.



$OM \rightarrow$ consumer is willing to pay
 $OP_0 \rightarrow$ Actual price in the market
 $P_0CM \rightarrow$ consumer surplus.

$$C.S = \int_0^{x_0} \text{demand function} \cdot dx - P \times Q.$$

Example

Demand function $P = 35 - 2x - x^2$ and the demand x_0 is 3.

$$\begin{aligned} P &= 35 - 2(3) - (3)^2 \\ &= 35 - 6 - 9 \\ &= 20. \end{aligned}$$

$$\begin{aligned} C.S &= \int_0^3 (35 - 2x - x^2) dx - (20 \times 3) \\ &= \left[35x - \frac{2x^2}{2} - \frac{x^3}{3} \right]_0^3 - 60 \\ &= 35(3) - (3)^2 - \frac{(3)^3}{3} - 60 \\ &= 105 - 9 - 9 - 60 \\ &= 27 \text{ units } \underline{\underline{Am}} \end{aligned}$$

Consumer Surplus under ~~perfect~~ ^{pure} Competition

Under pure competition price is determined by the demand and supply equilibrium.

Ex Given the functions of demand and supply

$$\text{Demand } P_D = \left(\frac{8}{x+1} - 2 \right)$$

$$\text{Supply } P_S = \left(\frac{x+3}{2} \right)$$

Equating $D = S$

$$\left(\frac{8}{x+1} - 2 \right) = \frac{x+3}{2}$$

$$\Rightarrow x^2 + 8x - 9 = 0$$

$$\Rightarrow x = 1 \text{ or } x = -9$$

$$\left[\frac{x+3}{2} = \frac{1+3}{2} = \frac{4}{2} = 2 \right]$$

$$A \quad x = 1 \quad p = 2$$

$$C.S = \int_0^1 \left(\frac{8}{x+1} - 2 \right) dx - (2 \times 1)$$

$$= \left[8 \log(x+1) - 2x \right]_0^1 - 2$$

$$= 8 \log 2 - 2 - 2$$

$$= 8(0.693) - 4$$

$$= 1.54 \text{ (Ans)}$$

Consumer Surplus under Monopoly

To determine the price under monopoly market we must equate the MR and MC.

$$\underline{\text{Ex}} \quad P_d = (6-x)^2 \quad P_s = 14+x$$

$$\begin{aligned} TR &= P_d \times x \\ &= (6-x)^2 \cdot x \\ &= (36 - 12x + x^2) x \\ &= 36x - 12x^2 + x^3 \end{aligned}$$

$$MR = \frac{dR}{dx} = 36 - 24x + 3x^2$$

Supply Price $P_s = 14+x = MC$ (Supply price is to be identical with MC).

$$MC = MR$$

$$36 - 24x + 3x^2 = 14 + x$$

$$\Rightarrow (x+1)(3x-22) = 0$$

$$\Rightarrow x = -1 \text{ or } x = \frac{22}{3} = 7.33$$

$$\text{At } x=1 \quad P_d = 25 \quad \left[(6-x)^2 = 36 - 12x + x^2 = 36 - 12 + 1 = 25 \right]$$

$$C.S = \int_0^1 (36 - 12x + x^2) dx - (25 \times 1)$$

$$= 36x - \frac{12x^2}{2} + \frac{x^3}{3} - 25$$

$$= 36 - 6 + \frac{1}{3} - 25$$

$$= \frac{16}{3} \text{ mit } \underline{\underline{Ans}}$$